Herr DDDr. Franz Langmayr hat im Jahre 2005 eine Arbeit über "Antinomie im Zermelo-Fraenkel-Axiomensystem" geschrieben, deren Gedankengänge in gleicher Weise wie die seiner Arbeit zum Auswahlpostulat berichtenswert erscheinen.

Antinomial Predicates in ZFC

Does the Zermelo-Fraenkel-Choice (ZFC) system of axioms prevent antinomies? None that I know, ever claimed it. Some may have hoped for it.

Here is the easy construction of an antinomial predicate Q.

(1) Let Q(x) mean that x be a predicate, such that x(x) = false.

Then (2) from Q(Q) = true follows Q(Q) = false.

And (3) from Q(Q) = false follows Q(Q) = true.

An antinomy.

In this construction we assume that predicates can be subject to predicates. ZFC axioms do not prevent that.

Going more deeply into (3) we can say that from

Q(Q) = false

follows that either Q is not a predicate, though is has been defined as such, or that Q(Q) is not false.

So if we allow Q to be a predicate according to its definition, the conclusion

Q(Q) != false, i.e. Q(Q) = true

is immediate.

Yet it would be legitimate to conclude from the contradiction that (1) is not allowed as a definition of a predicate.

This however does not follow from any ZFC axiom since only the Axiom of Separation is making use of predicate notion, yet does not involve restrictions on predicates at all.

So our result may have us reformulate the Axiom of Separation in order to exclude antinomious predicates such as Q.

But honestly, I do not think that a useful and totally unantinomious system of axioms does exist. Neither should „good“ mathematicians have a duty to sweep antinomies under a carpet.

I do think that any tool – physical or mathematical – is bound to be limited and that we just should be aware of these limitations.

Literature

Robert L. Vaught: Set Theory, An Introduction, 2. edition 1995, chapter 6.

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